

Alternating Current — Assignment 2

Class 12 Physics · Higher-difficulty challenge set + integrated problems · Answer key included · Narayan Gurukul Academy

Instructions: (1) Attempt all questions; start with the topic-wise challenge set, then attempt the integrated (multi-topic) problems.
(2) Give all answers with correct SI units. $g = 10 \text{ m/s}^2$ unless stated.
(3) A final answer key is provided at the end — try the full set before checking it.

Part A — Topic-wise challenge questions

RMS and Average Values

1. Derive $I_{\text{rms}} = I_0/\sqrt{2}$ for an alternating current.
2. An AC has a peak value of 14.14 A. Find its RMS value.
3. Find the average value of an AC over a half cycle.

Reactance and Impedance

1. Define inductive and capacitive reactance and their frequency dependence.
2. A 0.1 H inductor is connected to a source of $\omega = 100 \text{ rad/s}$. Find X_L .
3. Derive the impedance of a series LCR circuit.

Resonance

1. Derive the condition for resonance in a series LCR circuit.
2. An LCR circuit has $L = 0.5 \text{ H}$ and $C = 20 \mu\text{F}$. Find its resonant frequency.
3. Define the quality factor Q and derive its expression.

Power and Power Factor

1. Derive the average power in a series LCR circuit.
2. An AC circuit has $\cos\phi = 0.8$, $V_{\text{rms}} = 200 \text{ V}$, $I_{\text{rms}} = 5 \text{ A}$. Find the average power.
3. Show that the average power in a pure inductor over a cycle is zero.

LC Oscillations

1. Explain the energy changes in an LC oscillator.
2. An LC circuit has $L = 2 \text{ mH}$ and $C = 8 \mu\text{F}$. Find the frequency of oscillation.
3. Show the analogy between an LC oscillator and a mechanical oscillator.

Part B — Integrated problems (multi-topic)

1. A series LCR circuit with $R = 30 \Omega$, $L = 0.5 \text{ H}$ and $C = 20 \mu\text{F}$ is connected to a 200 V, 50 Hz AC source. Find the impedance, the current, and the phase angle.
2. An AC circuit has $V = 200 \sin(100t) \text{ V}$ and a current $I = 5\sqrt{2} \sin(100t - \pi/4) \text{ A}$. Find the RMS voltage, RMS current, impedance, power factor and average power.

3. A coil of inductance 2 H and resistance 10 Ω is connected to a 200 V, 50 Hz supply. Find the impedance, current and power factor.
4. A 100 μF capacitor is connected in series with a 100 Ω resistor and a source of 100 V (RMS), 50 Hz. Find the capacitive reactance, the impedance and the current.
5. A series LCR circuit resonates at 1000 Hz. If its bandwidth is 100 Hz, find its quality factor, and state the voltage magnification across the inductor and capacitor.

Answer Key

RMS and Average Values

$I_{\text{rms}} = I_0/\sqrt{2}$; I_{avg} over a full cycle = 0; over a half cycle = $2I_0/\pi$. For $I_0 = 14.14$ A: $I_{\text{rms}} = 10$ A.

Reactance and Impedance

$X_L = \omega L$, $X_C = 1/\omega C$; for $L = 0.1$ H, $\omega = 100$: $X_L = 10 \Omega$. $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

Resonance

Resonance when $X_L = X_C$, $\omega_0 = 1/\sqrt{LC}$. For $L = 0.5$ H, $C = 20 \mu\text{F}$: $\omega_0 \approx 316$ rad/s, $f_0 \approx 50.3$ Hz. $Q = \omega_0 L/R = 1/(\omega_0 CR)$.

Power and Power Factor

$P = V_{\text{rms}} I_{\text{rms}} \cos\phi$. For 200 V, 5 A, $\cos\phi = 0.8$: $P = 800$ W. Pure L or C: $\cos\phi = 0$, $P = 0$.

LC Oscillations

Energy alternates between $\frac{1}{2}q^2/C$ and $\frac{1}{2}Li^2$; total $\frac{1}{2}q_0^2/C = \frac{1}{2}Li_0^2$ is constant. $\nu = 1/2\pi\sqrt{LC} \approx 1258$ Hz for $L = 2$ mH, $C = 8 \mu\text{F}$.

Integrated problems

1: $X_L \approx 157 \Omega$, $X_C \approx 159 \Omega$, $Z \approx 30 \Omega$, $I \approx 6.7$ A, $\phi \approx 0^\circ$. 2: $V_{\text{rms}} = 141$ V, $I_{\text{rms}} = 5$ A, $Z = 28.3 \Omega$, $\cos\phi = 0.707$, $P = 500$ W. 3: $X_L = 628 \Omega$, $Z \approx 628 \Omega$, $I \approx 0.32$ A, $\cos\phi \approx 0.016$. 4: $X_C = 1/(2\pi \cdot 50 \cdot 100\mu\text{F}) \approx 31.8 \Omega$, $Z \approx 104.9 \Omega$, $I \approx 0.95$ A. 5: $Q = f_0/\Delta f = 10$; voltage across L and C is Q times the supply voltage.