

Application of Derivatives — Assignment 2

Class 12 Maths · Higher-difficulty challenge set + integrated problems · Answer key included · Narayan Gurukul Academy

Instructions: (1) Attempt all questions; start with the topic-wise challenge set, then attempt the integrated (multi-topic) problems.
 (2) Give all answers with correct SI units. $g = 10 \text{ m/s}^2$ unless stated.
 (3) A final answer key is provided at the end — try the full set before checking it.

Part A — Topic-wise challenge questions

Rate of Change and Tangents

- Find the slope of the tangent to $y = x^2$ at $x = 1$.
- The radius of a circle increases at 0.1 cm/s . Find the rate of increase of area when $r = 5 \text{ cm}$.
- The volume of a cube is increasing at $9 \text{ cm}^3/\text{s}$. Find the rate of increase of its side when the side is 10 cm .

Increasing and Decreasing Functions

- Find the interval in which $f(x) = x^2 - 4x + 3$ is increasing.
- Show that $f(x) = x^3$ is increasing on $\mathbb{R}$.
- Find the intervals in which $f(x) = x^3 - 3x^2 - 9x + 5$ is increasing or decreasing.

Maxima and Minima

- Find the local maximum and minimum of $f(x) = x^2 - 4x + 3$.
- Find the critical points of $f(x) = x^3 - 3x$.
- Find the local maxima and minima of $f(x) = 2x^3 - 3x^2 - 12x + 5$.

Optimisation and Approximations

- Use linear approximation to find $(1.01)^3$.
- Find the maximum area of a rectangle with perimeter 20.
- Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

Part B — Integrated problems (multi-topic)

- Find the slope of the tangent to $y = x^2$ at $x = 1$.
- Find the interval in which $f(x) = x^2 - 4x + 3$ is increasing.
- Find the local maximum and minimum of $f(x) = x^2 - 4x + 3$.
- Use linear approximation to find $(1.01)^3$.
- Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

Answer Key

Rate of Change and Tangents

Slope = 2. $dA/dt = 2\pi r(dr/dt) = \pi \approx 3.14 \text{ cm}^2/\text{s}$. $dV/dt = 3a^2(da/dt)$ $\therefore da/dt = 9/(3 \cdot 100) = 0.03 \text{ cm/s}$.

Increasing and Decreasing Functions

Increasing on $(2, \infty)$. x^3 increasing on $\mathbb{R}$. f increasing on $(-\infty, -1)$ and $(3, \infty)$, decreasing on $(-1, 3)$.

Maxima and Minima

Minimum at $x = 2$. Critical points $x = \pm 1$. Local max at $x = -1$, local min at $x = 2$.

Optimisation and Approximations

1.03. Max area = 25 (square of side 5). Numbers 7.5 and 7.5.

Integrated problems

1: 2. 2: $(2, \infty)$. 3: min at $x = 2$. 4: 1.03. 5: 7.5 and 7.5.