

Application of Derivatives — CBSE Pattern Test 2

Class 12 Maths · 90 minutes · 40 marks · Sections A–D with answer key · Narayan Gurukul Academy

Test 2 — CBSE Pattern (90 minutes, 40 marks)

Class 12 Maths · Application of Derivatives · Narayan Gurukul Academy (ClassApna)

General Instructions: (1) This paper has four sections — Section A (1×10), Section B (2×4), Section C (3×4) and Section D (5×2) — total 40 marks. Time allowed: 90 minutes.

(2) All questions are compulsory. No overall choice is given; internal choice, where allowed, is clearly marked.

(3) For MCQ questions choose the single correct option; for VSA answers write a single line.

(4) Take $g = 10 \text{ m/s}^2$. All symbols carry their usual meaning. Use of a calculator is not permitted.

Section A — MCQ & VSA ($10 \times 1 = 10$ marks)

- The rate of change of y with respect to x is
 - dy/dx
 - dx/dy
 - xy
 - $x + y$
- If the radius of a circle increases, the rate of change of area is
 - $2\pi r(dr/dt)$
 - πr^2
 - $2\pi r$
 - $\pi r(dr/dt)$
- The critical points are where
 - $f'(x) = 0$ or f' is undefined
 - $f(x) = 0$
 - $f''(x) = 0$
 - $x = 0$
- For x^2 on $\mathbb{R}$, the function is
 - decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$
 - always increasing
 - always decreasing
 - constant
- The second-derivative test uses
 - $f''(x)$ at the critical point
 - $f(x)$ only
 - the y -intercept
 - the domain
- The local maximum of $f(x) = -x^2$ is at
 - $x = 0$
 - $x = 1$
 - $x = -1$
 - $x = 2$
- The absolute maximum of a continuous function on $[a, b]$ occurs at
 - a critical point or an endpoint

- (B) only a critical point
 (C) only the midpoint
 (D) always the endpoint
8. The approximate value of $(1.01)^3$ using linear approximation is
 (A) 1.03
 (B) 1.01
 (C) 1.3
 (D) 1.003
9. Define the rate of change of a quantity.
10. Write the linear approximation formula.

Section B — Short Answer I ($4 \times 2 = 8$ marks)

11. State the condition for a function to be decreasing.
12. State the condition for a local minimum.
13. Find the critical point of $f(x) = x^2 - 2x$.
14. Where does the absolute maximum of f on $[a, b]$ occur?

Section C — Short Answer II ($4 \times 3 = 12$ marks)

15. The radius of a circle increases at 0.1 cm/s. Find the rate of increase of area when $r = 5$ cm.
16. Show that $f(x) = x^3$ is increasing on $\mathbb{R}$.
17. Find the critical points of $f(x) = x^3 - 3x$.
18. Find the maximum area of a rectangle with perimeter 20.

Section D — Long Answers ($2 \times 5 = 10$ marks)

19. Find the intervals in which $f(x) = 2x^3 - 15x^2 + 36x + 1$ is increasing or decreasing.
20. Find the maximum and minimum values of $f(x) = x^3 - 6x^2 + 9x + 15$. A wire of length 28 m is to be cut into two pieces to form a square and a circle. Find the lengths so that the total area enclosed is minimum.

Answer Key (Marking Scheme)

- Q1 — (A) dy/dx . Q2 — (A) $2\pi r(dr/dt)$. Q3 — (A) $f'(x) = 0$ or f' undefined. Q4 — (A) decreasing on $(-\infty, 0)$, increasing on $(0, \infty)$.
- Q5 — (A) $f''(x)$ at the critical point. Q6 — (A) $x = 0$. Q7 — (A) a critical point or an endpoint. Q8 — (A) 1.03.
- Q9 — dy/dx measures the rate of change.
- Q10 — $f(x + \Delta x) \approx f(x) + f'(x)\Delta x$.
- Q11 — $f'(x) \leq 0$.
- Q12 — $f'(x) = 0$ and $f''(x) > 0$.
- Q13 — $x = 1$.
- Q14 — At a critical point or an endpoint.
- Q15 — $\pi \approx 3.14 \text{ cm}^2/\text{s}$.
- Q16 — $f'(x) = 3x^2 \geq 0$.
- Q17 — $x = \pm 1$.

Q18 — 25.

Q19 — Increasing on $(-\infty, 2) \cup (3, \infty)$; decreasing on $(2, 3)$.

Q20 — Max 15 at $x = 0$? No: max at $x = 1$, min at $x = 3$; side lengths ratio $2 : \pi$ interpretation as in notes.