

Binomial Theorem — Assignment 2

Class 11 Maths · Higher-difficulty challenge set + integrated problems · Answer key included · Narayan Gurukul Academy

Instructions: (1) Attempt all questions; start with the topic-wise challenge set, then attempt the integrated (multi-topic) problems.
 (2) Give all answers with correct SI units. $g = 10 \text{ m/s}^2$ unless stated.
 (3) A final answer key is provided at the end — try the full set before checking it.

Part A — Topic-wise challenge questions

Binomial Expansion for Positive Integral Index

1. Expand $(1 + x)^4$ using the binomial theorem.
2. Expand $(2x + 1)^3$ using the binomial theorem.
3. Using the binomial theorem, expand $(2 + 3x)^4$ and find the coefficient of x^2 .

Pascal's Triangle

1. Write the expansion of $(a + b)^4$ using Pascal's triangle.
2. Explain how Pascal's triangle gives the binomial coefficients.
3. Using Pascal's triangle, expand $(1 + x)^5$ and write all the coefficients.

General Term

1. Find the coefficient of x^3 in the expansion of $(1 + x)^8$.
2. Find the 5th term in the expansion of $(x + 2)^6$.
3. Find the term independent of x in the expansion of $(x - 2/x^2)^9$.

Middle Term

1. Find the middle term in the expansion of $(x + 1/x)^6$.
2. Find the two middle terms in the expansion of $(a + b)^7$.
3. Find the middle term in the expansion of $(2x + 1/x^2)^8$.

Properties of Binomial Coefficients

1. Prove that $C(n,0) + C(n,1) + \dots + C(n,n) = 2^n$.
2. Find the sum of the even-position and odd-position coefficients in $(1 + x)^n$.
3. Prove that $C(n,1) + C(n,3) + \dots = 2^{n-1}$.

Applications and Simplifications

1. Using the binomial theorem, evaluate $(1.01)^5$ to three decimal places.
2. Find the value of $(2 + \sqrt{3})^4 - (2 - \sqrt{3})^4$.
3. Prove that $(\sqrt{2} + 1)^5 - (\sqrt{2} - 1)^5 = 82\sqrt{2}$.

Part B — Integrated problems (multi-topic)

1. Expand $(1 + x)^4$ using the binomial theorem.
2. Find the coefficient of x^5 in $(1 + x)^{10}$.
3. Find the middle term in the expansion of $(x + 1/x)^6$.
4. Prove that $C(n,0) + C(n,1) + \dots + C(n,n) = 2^n$.
5. Using the binomial theorem, evaluate $(1.01)^5$ to three decimal places.

Answer Key**Binomial Expansion for Positive Integral Index**

$(1 + x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$. $(2x + 1)^3 = 8x^3 + 12x^2 + 6x + 1$. $(2 + 3x)^4$: coefficient of $x^2 = 6 \times 2^2 \times 3^2 = 216$.

Pascal's Triangle

Row $n = 4$: 1, 4, 6, 4, 1 gives coefficients. $(1 + x)^5$ coefficients: 1, 5, 10, 10, 5, 1.

General Term

Coefficient of x^3 in $(1+x)^8 = 8C3 = 56$. 5th term of $(x+2)^6 = 6C4 x^2 2^4 = 240x^2$. Term independent of x in $(x - 2/x^2)^9$: $r = 3$, term = $9C3(-2)^3 = -672$.

Middle Term

Middle term of $(x + 1/x)^6$ is the 4th term = $6C3 = 20$. Two middle terms of $(a + b)^7$ are the 4th and 5th. Middle term of $(2x + 1/x^2)^8$ is the 5th term.

Properties of Binomial Coefficients

Sum = 2^n . Even and odd coefficient sums are each 2^{n-1} . $C(n,1) + C(n,3) + \dots = 2^{n-1}$.

Applications and Simplifications

$(1.01)^5 \approx 1.051$. $(2+\sqrt{3})^4 - (2-\sqrt{3})^4 = 4 \cdot [\text{odd terms}] = 8 \times (2^3\sqrt{3} + \dots) = 96\sqrt{3}$. $(\sqrt{2}+1)^5 - (\sqrt{2}-1)^5 = 82\sqrt{2}$.

Integrated problems

1: $1 + 4x + 6x^2 + 4x^3 + x^4$. 2: 252. 3: 20. 4: Put $x = 1$ in $(1 + x)^n$. 5: 1.051.