

Complex Numbers and Quadratic Equations — Assignment 2

Class 11 Maths · Higher-difficulty challenge set + integrated problems · Answer key included · Narayan Gurukul Academy

Instructions: (1) Attempt all questions; start with the topic-wise challenge set, then attempt the integrated (multi-topic) problems.
(2) Give all answers with correct SI units. $g = 10 \text{ m/s}^2$ unless stated.
(3) A final answer key is provided at the end — try the full set before checking it.

Part A — Topic-wise challenge questions

The Imaginary Unit and Standard Form

- Express $(1 + i)(2 - i)$ in the standard form $a + ib$.
- Find the values of i^{-1} and i^{-2} .
- Express $i^9 + i^{19}$ and $(3 + i\sqrt{5})(3 - i\sqrt{5})$ in the form $a + ib$.

Addition, Subtraction, Multiplication and Division

- If $z_1 = 3 + 4i$ and $z_2 = 2 - i$, find $z_1 + z_2$ and $z_1 z_2$.
- Express $1/(1 + i)$ in the form $a + ib$.
- Express $(3 + 2i)/(1 - 2i)$ in the standard form $a + ib$.

Conjugate and Modulus

- Find the modulus and conjugate of $1 + i\sqrt{3}$.
- If $z = 2 + 3i$, verify that $z \cdot \bar{z} = |z|^2$.
- If $z_1 = 2 + i$ and $z_2 = 1 - 3i$, find $|z_1 z_2|$ and show that $|z_1 z_2| = |z_1| |z_2|$.

Argand Plane and Polar Form

- Find the modulus and argument of $-1 + i\sqrt{3}$.
- Represent $2 + 2i$ in polar form.
- Find the modulus and argument of $z = -1 - i$ and express it in polar form.

Quadratic Equations with Complex Roots

- Solve $x^2 + 2 = 0$.
- Solve the quadratic equation $x^2 - 4x + 13 = 0$.
- Solve $x^2 + x + 1 = 0$ and write the roots in the form $a + ib$.

Cube Roots of Unity

- Prove that $1 + \omega + \omega^2 = 0$.
- If ω is a cube root of unity, find the value of $\omega^{10} + \omega^{20}$.
- If ω is a non-real cube root of unity, prove that $(1 + \omega)^3 - (1 + \omega^2)^3 = 0$.

Part B — Integrated problems (multi-topic)

- Express $(1 + i)(2 - i)$ in the standard form $a + ib$.
- Express $(3 + 2i)/(1 - 2i)$ in the standard form $a + ib$.
- Find the modulus and argument of $z = -1 - i$ and express it in polar form.
- Solve the quadratic equation $x^2 - 4x + 13 = 0$.
- If ω is a cube root of unity, prove that $1 + \omega + \omega^2 = 0$.

Answer Key**The Imaginary Unit and Standard Form**

$$(1 + i)(2 - i) = 3 + i, i^{-1} = -i, i^{-2} = -1, i^9 + i^{19} = i + (-i) = 0; (3 + i\sqrt{5})(3 - i\sqrt{5}) = 9 + 5 = 14.$$

Addition, Subtraction, Multiplication and Division

$$z_1 + z_2 = 5 + 3i, z_1 z_2 = 10 + 5i, 1/(1 + i) = (1 - i)/2, (3 + 2i)/(1 - 2i) = (-1 + 8i)/5.$$

Conjugate and Modulus

$$|1 + i\sqrt{3}| = 2, \text{ conjugate} = 1 - i\sqrt{3}, z\bar{z} = 13 = |z|^2, |z_1 z_2| = |z_1||z_2| = \sqrt{5} \times \sqrt{10} = \sqrt{50}.$$

Argand Plane and Polar Form

$$-1 + i\sqrt{3}: \text{modulus } 2, \text{ argument } 2\pi/3, 2 + 2i = 2\sqrt{2}(\cos \pi/4 + i \sin \pi/4), -1 - i: \text{modulus } \sqrt{2}, \text{ argument } -3\pi/4, \text{ polar form } \sqrt{2}(\cos(-3\pi/4) + i \sin(-3\pi/4)).$$

Quadratic Equations with Complex Roots

$$x^2 + 2 = 0 \Rightarrow x = \pm i\sqrt{2}, x^2 - 4x + 13 = 0 \Rightarrow x = 2 \pm 3i, x^2 + x + 1 = 0 \Rightarrow x = (-1 \pm i\sqrt{3})/2.$$

Cube Roots of Unity

$$1, \omega, \omega^2 \text{ with } \omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0, \omega^{10} + \omega^{20} = \omega + \omega^2 = -1, (1 + \omega)^3 - (1 + \omega^2)^3 = 0.$$

Integrated problems

$$1: 3 + i, 2: (-1 + 8i)/5, 3: \sqrt{2}(\cos(-3\pi/4) + i \sin(-3\pi/4)), 4: 2 \pm 3i, 5: \text{Proof via } \omega^3 = 1.$$