

Continuity and Differentiability — Assignment 2

Class 12 Maths · Higher-difficulty challenge set + integrated problems · Answer key included · Narayan Gurukul Academy

Instructions: (1) Attempt all questions; start with the topic-wise challenge set, then attempt the integrated (multi-topic) problems.
 (2) Give all answers with correct SI units. $g = 10 \text{ m/s}^2$ unless stated.
 (3) A final answer key is provided at the end — try the full set before checking it.

Part A — Topic-wise challenge questions

Continuity of a Function

1. Check the continuity of $f(x) = x^2$ at $x = 2$.
2. Discuss the continuity of $f(x) = 1/(x - 1)$ at $x = 1$.
3. Find the value of k so that $f(x) = kx + 1$ for $x \leq 2$ and $3x - 1$ for $x > 2$ is continuous at $x = 2$.

Differentiability

1. Check the differentiability of $f(x) = |x|$ at $x = 0$.
2. Find the derivative of $f(x) = x^3$ from first principles.
3. Show that $f(x) = |x|$ is continuous but not differentiable at $x = 0$.

Rules of Differentiation

1. Differentiate $x^3 + 2x$ with respect to x .
2. Differentiate $x \sin x$ with respect to x .
3. Differentiate $x^2 \sin x$ using the product rule.

Logarithmic, Implicit and Parametric Differentiation

1. Differentiate x^x with respect to x .
2. Find dy/dx if $x^2 + y^2 = 25$.
3. Find dy/dx if $x = a \cos \theta$, $y = a \sin \theta$.

Part B — Integrated problems (multi-topic)

1. Check the continuity of $f(x) = x^2$ at $x = 2$.
2. Does differentiability imply continuity?
3. Write the derivative of x^n .
4. Find dy/dx if $x^2 + y^2 = 25$.
5. Differentiate $x^3 + 2x$ with respect to x .

Answer Key

Continuity of a Function

Continuous since $\lim_{x \rightarrow 2} f(x) = f(2) = 4$. Discontinuous at $x = 1$. $k: 2k + 1 = 5 \Rightarrow k = 2$.

Differentiability

Not differentiable at $x = 0$. $f'(x) = 3x^2$. Continuous but the left and right derivatives differ at 0.

Rules of Differentiation

$$3x^2 + 2 \cdot \cos x \cdot x + \sin x = x \cos x + \sin x \cdot 2x \sin x + x^2 \cos x.$$

Logarithmic, Implicit and Parametric Differentiation

$$x^x(1 + \ln x). \frac{dy}{dx} = -x/y. \frac{dy}{dx} = (a \cos \theta)/(-a \sin \theta) = -\cot \theta.$$

Integrated problems

1: continuous. 2: yes. 3: $n x^{n-1}$. 4: $-x/y$. 5: $3x^2 + 2$.