

Determinants — Assignment 2

Class 12 Maths · Higher-difficulty challenge set + integrated problems · Answer key included · Narayan Gurukul Academy

Instructions: (1) Attempt all questions; start with the topic-wise challenge set, then attempt the integrated (multi-topic) problems.
 (2) Give all answers with correct SI units. $g = 10 \text{ m/s}^2$ unless stated.
 (3) A final answer key is provided at the end — try the full set before checking it.

Part A — Topic-wise challenge questions

Expansion of Determinants

1. Evaluate $|\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}|$.
2. Find the value of $|\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}|$.
3. Evaluate the determinant $|\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}|$.

Properties of Determinants

1. Prove that $|\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}| = 0$ using properties.
2. Evaluate $|\begin{bmatrix} a & b \\ b & a \end{bmatrix}|$.
3. Show that $|\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}| = (a - b)(b - c)(c - a)$.

Adjoint, Inverse and Singular Matrices

1. Find the adjoint of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.
2. Find $|A|$ and state whether $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is invertible.
3. Find A^{-1} for $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.

Solving Linear Equations with Cramer's Rule

1. Solve $x + y = 5$, $x - y = 1$ using determinants.
2. Solve $2x + y = 4$, $x + y = 3$ by Cramer's rule.
3. Solve $2x + 3y = 5$, $x - y = 1$ using Cramer's rule.

Area of a Triangle

1. Find the area of the triangle with vertices $(0, 0)$, $(1, 0)$, $(0, 1)$.
2. Find the area of the triangle with vertices $(1, 1)$, $(2, 2)$, $(3, 3)$ and comment.
3. Find the area of the triangle with vertices $(1, 0)$, $(2, 2)$, $(0, 3)$.

Part B — Integrated problems (multi-topic)

1. Evaluate $|\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}|$.
2. Find the adjoint of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.
3. Solve $x + y = 5$, $x - y = 1$ using determinants.
4. Find the area of the triangle with vertices $(0, 0)$, $(1, 0)$, $(0, 1)$.
5. Solve $2x + y = 4$, $x + y = 3$ by Cramer's rule.

Answer Key

Expansion of Determinants

−2. 10. 0.

Properties of Determinants

0. $a^2 - b^2$. $(a - b)(b - c)(c - a)$.

Adjoint, Inverse and Singular Matrices

$\text{adj}(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$. $|A| = -2$ (invertible). $A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$.

Solving Linear Equations with Cramer's Rule

$x = 3, y = 2$. $x = 1, y = 2$. $x = 8/5, y = 3/5$.

Area of a Triangle

$1/2$. 0 (collinear). $5/2$.

Integrated problems

1: −2. 2: $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$. 3: $x = 3, y = 2$. 4: $1/2$. 5: $x = 1, y = 2$.