

Electrostatic Potential and Capacitance — Assignment 2

Class 12 Physics · Higher-difficulty challenge set + integrated problems · Answer key included · Narayan Gurukul Academy

Instructions: (1) Attempt all questions; start with the topic-wise challenge set, then attempt the integrated (multi-topic) problems.
(2) Give all answers with correct SI units. $g = 10 \text{ m/s}^2$ unless stated.
(3) A final answer key is provided at the end — try the full set before checking it.

Part A — Topic-wise challenge questions

Electric Potential and Potential Difference

1. Derive the relation between electric field and electric potential.
2. Two point charges $+4 \mu\text{C}$ and $-4 \mu\text{C}$ are separated by 0.2 m . Find the potential at the midpoint.
3. Calculate the work done in moving a charge of 2 C through a potential difference of 12 V .

Capacitance and Capacitors

1. Derive the expression for the capacitance of a parallel-plate capacitor.
2. A parallel-plate capacitor has plates of area 0.02 m^2 separated by 1 mm . Find its capacitance in air.
3. A $5 \mu\text{F}$ capacitor is charged to 100 V . Find the charge and the energy stored.

Dielectrics and Polarisation

1. Explain why the capacitance increases when a dielectric fills a capacitor.
2. Derive the capacitance of a parallel-plate capacitor partly filled with a dielectric slab of thickness t and dielectric constant K .
3. Distinguish between polar and non-polar molecules and explain polarisation.

Combinations of Capacitors

1. Two capacitors of $2 \mu\text{F}$ and $3 \mu\text{F}$ are joined (i) in series and (ii) in parallel. Find the equivalent capacitance.
2. Derive the equivalent capacitance of three capacitors in series.
3. A $1 \mu\text{F}$ and a $2 \mu\text{F}$ capacitor are in series, and this combination is in parallel with a $3 \mu\text{F}$ capacitor. Find the total capacitance.

Energy Stored in a Capacitor

1. Derive the expression $U = \frac{1}{2}CV^2$ for the energy stored in a capacitor.
2. A $10 \mu\text{F}$ capacitor is charged to 50 V . Find the energy stored.
3. Derive the energy density of an electric field, $\frac{1}{2}\epsilon_0 E^2$.

Part B — Integrated problems (multi-topic)

1. A parallel-plate capacitor of area 100 cm^2 and plate separation 2 mm is charged by a 100 V battery. Find its capacitance, the charge and the energy stored, and then the energy if the separation is doubled with the battery disconnected.

2. Three capacitors of 1 μF , 2 μF and 3 μF are connected in series across a 12 V battery. Find the equivalent capacitance, the charge on each and the potential difference across the 2 μF capacitor.
3. A parallel-plate capacitor is half filled with a dielectric of dielectric constant 5 parallel to the plates. If the original capacitance in air is C , find the new capacitance.
4. An isolated 4 μF capacitor is charged to 200 V and then disconnected. A dielectric of dielectric constant 2 is now inserted, completely filling it. Find the new capacitance, the new potential difference and the change in stored energy.
5. A charged capacitor of 5 μF at 100 V is connected across an uncharged 10 μF capacitor. Find the common potential and the energy lost in the process.

Answer Key

Electric Potential and Potential Difference

$E = -dV/dr$. At the midpoint, potentials cancel: $V = 0$. $W = qV = 2 \times 12 = 24 \text{ J}$.

Capacitance and Capacitors

$C = \epsilon_0 A/d \approx 1.77 \times 10^{-10} \text{ F}$. For 5 μF at 100 V: $Q = CV = 5 \times 10^{-4} \text{ C}$, $U = \frac{1}{2}CV^2 = 2.5 \times 10^{-2} \text{ J}$.

Dielectrics and Polarisation

The dielectric polarises, setting up an opposing field, so V falls and $C = Q/V$ rises by the factor K . Partly filled: $C = \epsilon_0 A/(d - t + t/K)$.

Combinations of Capacitors

(i) Series: $1/C = 1/2 + 1/3 \Rightarrow C = 1.2 \mu\text{F}$. (ii) Parallel: $C = 5 \mu\text{F}$. Series rule: $1/C = \sum 1/C_i$. Mixed: series part = $2/3 \mu\text{F}$, plus 3 μF in parallel $\approx 3.67 \mu\text{F}$.

Energy Stored in a Capacitor

$U = \frac{1}{2}CV^2 = \frac{1}{2}Q^2/C = \frac{1}{2}QV$. For 10 μF at 50 V: $U = 1.25 \times 10^{-2} \text{ J}$. Energy density $u = \frac{1}{2}\epsilon_0 E^2$.

Integrated problems

1: $C \approx 4.4 \times 10^{-11} \text{ F}$; $Q \approx 4.4 \times 10^{-9} \text{ C}$; $U \approx 2.2 \times 10^{-7} \text{ J}$; doubling d at constant Q doubles U . 2: $C = 6/11 \mu\text{F}$; $Q = C \times 12 \approx 6.5 \mu\text{C}$ on each; V across 2 $\mu\text{F} \approx 3.3 \text{ V}$. 3: $C' = \epsilon_0 A/d \times (K+1)/2 \times 2/\dots$, new $C = 2K/(K+1) \cdot C = 10/6 \text{ C} \approx 1.67 \text{ C}$. 4: $C' = 8 \mu\text{F}$; $V' = 100 \text{ V}$; $\Delta U = \frac{1}{2}CV^2(1/K - 1) = -0.04 \text{ J}$ (halved). 5: $V = 5 \times 100/15 \approx 33.3 \text{ V}$; $\Delta U \text{ lost} = \frac{1}{2}(5 \times 10)(100 - 33.3) \times 10^{-6} \approx 1.67 \times 10^{-2} \text{ J}$.