Gravitation — Assignment 2 (Higher Difficulty)

Class 11 Physics · NCERT Chapter 7

Narayan Gurukul Academy (ClassApna)

Instructions

- Multi-concept and JEE-style problems. Attempt all questions.
- Marks are indicated. Show complete reasoning.
- Use $G = 6.67 \times 10^{-11}\ \mathrm{N\ m^2 kg^{-2}}$, $M_e = 6 \times 10^{24}\ \mathrm{kg}$, $R_e = 6.4 \times 10^{6}\ \mathrm{m}$, $g = 9.8\ \mathrm{m\ s^{-2}}$.
- Time: 90 minutes.

Section A — MCQs (Multi-concept, 1 mark each)

1. A particle is projected upward from the surface of the Earth. If the speed of projection is $v_e/\sqrt{2}$ (where v_e is the escape velocity), the maximum height reached is —

- (a) R
- (b) $2R$
- (c) $R/2$
- (d) $R\sqrt{2}$

2. The work done in slowly lifting a body of mass m from the Earth's surface to a height equal to R (radius of Earth) is —

- (a) $\frac{GMm}{2R}$
- (b) $\frac{GMm}{R}$
- (c) $\frac{GMm}{3R}$
- (d) mgR

3. A satellite moves in an elliptical orbit around the Earth. The ratio of its speed at perigee to its speed at apogee is $2:1$. The ratio of the distances of perigee and apogee from the Earth's centre is —

- (a) 1 : 2
- (b) 2 : 1
- (c) 1 : 4
- (d) 4 : 1

4. The gravitational force between two point masses is F when they are a distance r apart. If one mass is tripled, the other is halved, and the distance is doubled, the new force is —

- (a) $\frac{3F}{8}$
- (b) $\frac{3F}{4}$
- (c) $\frac{3F}{2}$
- (d) $6F$

5. A body is weighed at the equator by a spring balance. If the Earth stops rotating suddenly, the reading of the balance will —

- (a) increase
- (b) decrease
- (c) remain unchanged
- (d) become zero

6. The time period of a satellite in a circular orbit of radius r is T. If its orbital radius is increased to $8r$, the new time period is —

- (a) $8T$
- (b) $16\sqrt{2}T$
- (c) $2\sqrt{2}T$
- (d) T

7. If g is the acceleration due to gravity on the Earth's surface, the gain in potential energy of an object of mass m raised from the surface to a height equal to the radius R of the Earth is —

- (a) $\frac{1}{2}mgR$
- (b) $\frac{1}{3}mgR$
- (c) $\frac{2}{3}mgR$
- (d) mgR

8. Two uniform solid spheres of radii R and $2R$ have the same density. The ratio of their escape velocities is —

- (a) $1:2$
- (b) $1:\sqrt{2}$
- (c) $1:4$
- (d) $2:1$

9. The gravitational potential difference between the surface of a planet of radius R and a point at height $h = R$ above its surface is —

- (a) $\frac{GM}{2R}$

- (b) $\frac{GM}{R}$
- (c) $\frac{GM}{3R}$
- (d) $\frac{2GM}{R}$

10. A body of mass m is taken from the surface of the Earth to a point at a distance equal to $3R$ from the Earth's centre through a perfectly smooth tunnel. The work done by gravity is —

- (a) $+\frac{2GMm}{3R}$
- (b) $-\frac{2GMm}{3R}$
- (c) $+\frac{GMm}{R}$
- (d) $-\frac{GMm}{3R}$

Section B — Assertion & Reason (1 mark each)

In each of the following, choose: (a) If both Assertion and Reason are true and the Reason correctly explains the Assertion. (b) If both are true but the Reason does not correctly explain the Assertion. (c) If the Assertion is true but the Reason is false. (d) If the Assertion is false but the Reason is true.

11. Assertion: The acceleration due to gravity at the centre of the Earth is zero.
Reason: The gravitational field inside a uniform spherical shell is zero.

12. Assertion: The orbital velocity of a satellite is independent of its mass.
Reason: The gravitational force provides the centripetal force and both are proportional to the satellite's mass.

13. Assertion: A geostationary satellite cannot be placed over a city like Delhi.
Reason: A geostationary satellite must orbit in the equatorial plane.

14. Assertion: Escape velocity is independent of the direction of projection.
Reason: Gravitational potential energy depends only on position, not on path.

Section C — Short Answer / Derivations (3 marks each)

15. Derive the relation between gravitational field intensity E and gravitational potential V. Hence find the field intensity at a point where the potential varies linearly with distance.

16. Derive an expression for the escape velocity by applying the law of conservation of mechanical energy, and show that for a projectile the escape velocity is $\sqrt{2}\,v_o$, where v_o is the orbital velocity of a near-surface satellite.

17. Using the shell theorem, derive the gravitational field intensity at (a) a point outside, (b) a point on the surface, and (c) a point inside a uniform solid sphere.

18. Show that the total mechanical energy of a satellite in a circular orbit is negative and equal to $-\frac{GMm}{2r}$, and interpret the negative sign as binding energy.

Section D — Numerical / Multi-topic Problems (5 marks each)

19. A satellite is revolving around the Earth in a circular orbit of radius $2R_e$. ($R_e = 6.4 \times 10^6$ m, $M_e = 6 \times 10^{24}$ kg)
(a) Calculate its orbital speed.
(b) Calculate the time period.
(c) What minimum additional kinetic energy per unit mass must be imparted to it so that it escapes the Earth's gravitational pull?

20. A planet of mass M_P has two moons. Moon A orbits at radius r with period T_A. Moon B orbits at radius $4r$.
(a) Using Kepler's third law, find T_B in terms of T_A.
(b) If the planet's radius is R_P and the acceleration due to gravity on its surface is g_P, express the escape velocity from the planet in terms of g_P and R_P.
(c) A projectile is launched from the planet's surface at speed $0.8\, v_{eP}$. Find the maximum height it reaches (assuming negligible atmosphere), in terms of R_P.

Answer Key — Assignment 2

- **Q1** — (a) Energies: total at surface $E = \frac{1}{2}m(v_e/\sqrt{2})^2 - \frac{GMm}{R} = -\frac{GMm}{2R}$; at height this equals $-\frac{GMm}{R+h}$; solving gives $h = R$
- **Q2** — (a) $\Delta U = \frac{GMm}{hR/(R+h)}$ evaluated with $h = R \rightarrow \frac{GMm}{2R}$
- **Q3** — (a) $v_p r_p = v_a r_a \rightarrow r_p : r_a = 1 : 2$
- **Q4** — (a) $F' = \frac{G(3m_1)(m_2/2)}{(2r)^2} = \frac{3F}{8}$
- **Q5** — (a) increase — the centrifugal term $mR\omega^2 \cos^2 \lambda$ disappears
- **Q6** — (b) $T \propto r^{3/2} \rightarrow T' = 8^{3/2} T = 16\sqrt{2}\, T$
- **Q7** — (a) $\Delta U = mgR \times \frac{h}{R+h} = \frac{1}{2}mgR$ at $h = R$
- **Q8** — (a) $v_e \propto R\sqrt{\rho}$ (since $M \propto R^3$) → ratio $1 : 2$
- **Q9** — (a) $V_h - V_R = -\frac{GM}{2R} + \frac{GM}{R} = \frac{GM}{2R}$
- **Q10** — (b) $W_g = U_i - U_f = -\frac{GMm}{R} + \frac{GMm}{3R} = -\frac{2GMm}{3R}$ (gravity does negative work; the lifting agent does $+\frac{2GMm}{3R}$)
- **Q11** — (a)
- **Q12** — (a)
- **Q13** — (a)
- **Q14** — (a)
- **Q15** — $E = -\frac{dV}{dr}$; if $V = kr$, then $E = -k$ (uniform field)

- **Q16** — Conserve energy: $\frac{1}{2}mv_e^2 - \frac{GMm}{R} = 0 \to v_e = \sqrt{2GM/R} = \sqrt{2}\sqrt{GM/R} = \sqrt{2}\,v_o$
- **Q17** — (a) GM/r^2 (b) GM/R^2 (c) GMr/R^3
- **Q18** — $K = \frac{GMm}{2r}, U = -\frac{GMm}{r} \to E = -\frac{GMm}{2r}$; negative energy binds the satellite
- **Q19** — (a) $v_o = \sqrt{GM/(2R_e)} \approx 5.59 \times 10^3$ m s^{-1} (b) $T = 2\pi\sqrt{(2R_e)^3/GM} = 4\pi\sqrt{2R_e/g} \approx 3.99$ h (c) escape needs $\Delta E/m = \frac{1}{2}v_e^2 - \frac{1}{2}v_o^2 = \frac{GM}{4R_e} \approx 1.56 \times 10^7$ J kg^{-1}
- **Q20** — (a) $T_B = 8T_A$ (b) $v_{eP} = \sqrt{2g_P R_P}$ (c) $h = R_P$ (conservation of energy gives $h = R_P \cdot \frac{v^2}{2g_P R_P - v^2}$ with $v = 0.8v_{eP} \to h = R_P$)