Gravitation — Test 1 (CBSE Pattern)

Class 11 Physics · NCERT Chapter 7

Time: 90 minutes · Maximum Marks: 40

Narayan Gurukul Academy (ClassApna)

General Instructions

1. All questions are compulsory.
2. Section A: Question 1–10 are MCQs/Very Short Answer type, each of **1 mark**.
3. Section B: Question 11–14 are of **2 marks** each.
4. Section C: Question 15–18 are of **3 marks** each.
5. Section D: Question 19–20 are of **5 marks** each.
6. Internal choice is provided in one question of 3 marks and one of 5 marks.
7. Use $\sqrt{2} = 1.41$, $\pi = 3.14$, $G = 6.67 \times 10^{-11}\ \mathrm{N\ m^2 kg^{-2}}$, $M_e = 6 \times 10^{24}\ \mathrm{kg}$, $R_e = 6.4 \times 10^6\ \mathrm{m}$, $g = 9.8\ \mathrm{m\ s^{-2}}$.

Section A — MCQ / VSA (10 × 1 = 10 marks)

1. The S.I. unit of gravitational potential is —

- (a) $\mathrm{J\ kg^{-1}}$
- (b) $\mathrm{N\ kg^{-1}}$
- (c) $\mathrm{J\ kg}$
- (d) $\mathrm{N\ m\ kg^{-1}}$

2. If the Earth's mass were doubled and its radius halved, the acceleration due to gravity on its surface would become —

- (a) $8g$
- (b) $4g$
- (c) $2g$
- (d) $g/2$

3. The ratio of the weight of a body at a height R above the surface to its weight on the surface is —

- (a) $1 : 4$

- (b) $1:2$
- (c) $4:1$
- (d) $2:1$

4. Why does the gravitational field inside a uniform spherical shell become zero? (VSA — state the theorem used.)

5. The escape velocity from the Moon's surface is **less than** the escape velocity from the Earth's surface. (State whether **True/False**.)

6. The orbital velocity of a satellite orbiting close to the Earth's surface is approximately —

- (a) 7.9 km s^{-1}
- (b) 11.2 km s^{-1}
- (c) 3.1 km s^{-1}
- (d) 1.4 km s^{-1}

7. Kepler's second law of planetary motion is a consequence of the conservation of —

- (a) angular momentum
- (b) linear momentum
- (c) energy
- (d) mass

8. A body of mass m is taken from a point at a distance R from the Earth's centre to a distance $2R$. The work done by the gravitational force is —

- (a) $\frac{GMm}{2R}$
- (b) $-\frac{GMm}{2R}$
- (c) $\frac{GMm}{R}$
- (d) $-\frac{GMm}{R}$

(Hint: $U(R) = -\frac{GMm}{R}$, $U(2R) = -\frac{GMm}{2R}$; work done by gravity $= U_i - U_f$.)

9. What is the period of revolution of a geostationary satellite? (VSA)

10. The total energy of a satellite in a circular orbit is negative because —

- (a) the satellite is bound to the Earth
- (b) kinetic energy is always negative
- (c) potential energy is zero
- (d) the orbit is elliptical

Section B — Short Answer I (4 × 2 = 8 marks)

11. Derive the expression for the gravitational field intensity at a point due to a point mass M.

12. Show that g at a height h above the Earth's surface (where $h \ll R$) is $g\left(1 - \frac{2h}{R}\right)$.

13. Write the expression for the gravitational potential energy of a system of two masses. Show that it is negative.

14. Distinguish between gravitational field intensity and gravitational potential.

Section C — Short Answer II (4 × 3 = 12 marks)

15. Derive the variation of acceleration due to gravity with depth below the Earth's surface, and plot g versus depth.

OR

State and prove Newton's shell theorem for a uniform spherical shell.

16. A satellite is orbiting the Earth at a height of 3600 km. Calculate its (a) orbital velocity and (b) time period. ($R_e = 6400$ km, $M_e = 6 \times 10^{24}$ kg)

17. Derive the expression for the escape velocity and show that $v_e = \sqrt{2}\, v_o$.

18. Using Kepler's third law, derive the relation $T^2 = \frac{4\pi^2}{GM} r^3$ for a satellite in a circular orbit.

Section D — Long Answer (2 × 5 = 10 marks)

19. (a) State Kepler's three laws of planetary motion.
(b) Prove that the line joining a planet to the Sun sweeps equal areas in equal times (Kepler's second law) using conservation of angular momentum.
(c) The mean distance of Mars from the Sun is 1.5 times that of the Earth from the Sun. Find the period of revolution of Mars around the Sun in years.

20. (a) Derive the total mechanical energy of a satellite in a circular orbit and show that it is $-\frac{GMm}{2r}$.
(b) Define binding energy of a satellite and calculate the binding energy per unit mass of a satellite orbiting at a height of 3600 km.
(c) If the radius of a planet is twice the Earth's radius and its density is half the Earth's density, calculate the ratio of the escape velocity from the planet to that from the Earth.

OR

(a) Derive the expression for the gravitational potential energy of a system of two masses.
(b) A body of mass 100 kg is lifted from the Earth's surface to a height equal to the Earth's radius. Calculate the work done.
(c) The centres of two spheres of masses 2 kg and 4 kg are 3 m apart. Find the gravitational potential at the midpoint of the line joining them.

Answer Key — Test 1

- **Q1** — (a) J kg^{-1}
- **Q2** — (a) $g' = G(2M)/(R/2)^2 = 8g$
- **Q3** — (a) $1:4$
- **Q4** — Newton's shell theorem
- **Q5** — True
- **Q6** — (a) 7.9 km s^{-1}
- **Q7** — (a) angular momentum
- **Q8** — (b) $W_g = U_i - U_f = -\frac{GMm}{R} + \frac{GMm}{2R} = -\frac{GMm}{2R}$
- **Q9** — 24 hours
- **Q10** — (a) the satellite is bound
- **Q11** — $E = GM/r^2$ directed towards the mass
- **Q12** — $g_h = g(1 + h/R)^{-2} \approx g(1 - 2h/R)$
- **Q13** — $U = -Gm_1m_2/r$
- **Q14** — E is a vector (force per unit mass, N kg^{-1}); V is a scalar (work per unit mass, J kg^{-1})
- **Q15** — $g_d = g(1 - d/R)$; graph is a straight line falling to zero at the centre
- **Q16** — $r = 10^7$ m; (a) $v_o = 6.33 \times 10^3$ m s^{-1} (b) $T \approx 2.76$ h
- **Q17** — $v_e = \sqrt{2GM/R} = \sqrt{2}(\sqrt{GM/R}) = \sqrt{2}\,v_o$
- **Q18** — Equate $GMm/r^2 = m(4\pi^2 r/T^2)$
- **Q19** — (c) $T_{Mars} \approx 1.84$ years
- **Q20** — (b) $BE/m = GM/2r \approx 2 \times 10^7$ J kg^{-1} (c) $1:1$