

Kinetic Theory — Assignment 2

Class 11 Physics · Higher-difficulty challenge set + integrated problems · Answer key included · Narayan Gurukul Academy

Instructions: (1) Attempt all questions; start with the topic-wise challenge set, then attempt the integrated (multi-topic) problems.
(2) Give all answers with correct SI units. $g = 10 \text{ m/s}^2$ unless stated.
(3) A final answer key is provided at the end — try the full set before checking it.

Part A — Topic-wise challenge questions

Ideal Gas and Its Assumptions

2 Mark Questions

1. What is an ideal gas? Under what conditions does a real gas approach ideal behaviour?
2. Find the number of moles in 44 g of CO_2 . (Molar mass = 44 g mol^{-1} .)

3 Mark Questions

1. A gas occupies 2 m^3 at a pressure of 10^5 Pa and a temperature of 300 K . Find the number of moles and the number of molecules. ($R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$, $N_A = 6.02 \times 10^{23}$.)

Pressure Exerted by an Ideal Gas

2 Mark Questions

1. The density of a gas is 1.5 kg m^{-3} and its molecules have a mean-square speed of $10^6 \text{ m}^2 \text{ s}^{-2}$. Find the pressure it exerts.

3 Mark Questions

1. A gas has a density of 2 kg m^{-3} and exerts a pressure of $2 \times 10^5 \text{ Pa}$. Find the root-mean-square speed of its molecules.

Kinetic Interpretation of Temperature

2 Mark Questions

1. Find the average translational kinetic energy of a molecule at 300 K . ($k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$.)

3 Mark Questions

1. Compute the total translational kinetic energy of 2 moles of an ideal gas at 400 K . ($R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$.)

Root-Mean-Square Speed

2 Mark Questions

1. Find the RMS speed of oxygen molecules at 300 K . ($M = 32 \times 10^{-3} \text{ kg mol}^{-1}$, $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$.)
2. If the temperature of a gas is raised from 300 K to 1200 K , by what factor does its RMS speed increase?

3 Mark Questions

1. At what temperature will the RMS speed of oxygen molecules equal that at which hydrogen has an RMS speed of 2000 m/s at 300 K ? Derive your answer.

Degrees of Freedom and Specific Heats

2 Mark Questions

- For a diatomic gas, using $f = 5$, find C_V , C_P and γ .

3 Mark Questions

- Compute the internal energy of 3 moles of a monatomic gas at 400 K. ($R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$.)

The Maxwell Speed Distribution

2 Mark Questions

- Find the most probable speed of nitrogen molecules at 300 K. ($M = 28 \times 10^{-3} \text{ kg mol}^{-1}$, $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$.)

3 Mark Questions

- Compute the most probable, mean and RMS speeds of argon atoms at 300 K. ($M = 40 \times 10^{-3} \text{ kg mol}^{-1}$, $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$.)

Part B — Integrated problems (multi-topic)

- A vessel holds 2 moles of oxygen gas ($M = 32 \text{ g/mol}$) at 300 K. Find (i) the total translational kinetic energy of the gas and (ii) the rms speed of its molecules. ($R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$.)
- A gas has a density of 1.6 kg/m^3 and exerts a pressure of $1.2 \times 10^5 \text{ Pa}$. Find the rms speed of its molecules.
- At a given temperature, by what factor is the rms speed of hydrogen molecules ($M = 2 \text{ g/mol}$) greater than that of oxygen molecules ($M = 32 \text{ g/mol}$)? Given that the rms speed of oxygen is 484 m/s at 300 K, find the rms speed of hydrogen at the same temperature.
- The rms speed of the molecules of a gas at 27°C is 500 m/s. At what temperature (in K and $^\circ \text{C}$) will their rms speed be doubled?
- Using the law of equipartition of energy, find C_V , C_P and γ for a monatomic gas.

Answer Key

Ideal Gas and Its Assumptions

2M(0): An ideal gas strictly obeys $PV = nRT$ at every pressure and temperature. A real gas approaches ideal behaviour at low pressure and high temperature, where the molecular volume and the inter-molecular forces become negligible. 2M(2): $n = m/M = 44/44 = 1 \text{ mole}$. 3M(1): $n = PV/RT = (10^5 \times 2)/(8.31 \times 300) \approx 80.2 \text{ mol}$; molecules = $n \cdot N_A = 80.2 \times 6.02 \times 10^{23} \approx 4.83 \times 10^{25}$.

Pressure Exerted by an Ideal Gas

2M(1): $P = (1/3)\rho v_{\text{rms}}^2 = (1/3) \times 1.5 \times 10^6 = 5 \times 10^5 \text{ Pa}$. 3M(1): $v_{\text{rms}} = \sqrt{3P/\rho} = \sqrt{(3 \times 2 \times 10^5)/2} = \sqrt{3 \times 10^5} \approx 548 \text{ m/s}$.

Kinetic Interpretation of Temperature

2M(1): $KE = (3/2)n k_B T = (3/2) \times 1.38 \times 10^{-23} \times 300 = 6.21 \times 10^{-21} \text{ J}$. 3M(1): total $KE = (3/2)nRT = (3/2) \times 2 \times 8.31 \times 400 = 9972 \text{ J} \approx 9.97 \text{ kJ}$.

Root-Mean-Square Speed

2M(0): $v_{rms} = \sqrt{3RT/M} = \sqrt{3 \times 8.31 \times 300/0.032} \approx 484 \text{ m/s}$. 2M(2): factor = $\sqrt{T_2/T_1} = \sqrt{1200/300} = 2$. 3M(1): $T = v^2 M/3R = (2000^2 \times 0.032)/(3 \times 8.31) \approx 5.13 \times 10^3 \text{ K}$.

Degrees of Freedom and Specific Heats

2M(2): $C_V = 5R/2 \approx 20.8 \text{ J mol}^{-1} \text{ K}^{-1}$, $C_P = 7R/2 \approx 29.1 \text{ J mol}^{-1} \text{ K}^{-1}$, $\gamma = C_P/C_V = 7/5 = 1.4$. 3M(1): internal energy $U = (3/2)nRT = (3/2) \times 3 \times 8.31 \times 400 \approx 1.5 \times 10^4 \text{ J}$.

The Maxwell Speed Distribution

2M(0): $v_{mp} = \sqrt{2RT/M} = \sqrt{2 \times 8.31 \times 300/0.028} \approx 422 \text{ m/s}$. 3M(1): $v_{mp} = \sqrt{2RT/M} \approx 353 \text{ m/s}$, $v_{mean} = \sqrt{8RT/\pi M} \approx 398 \text{ m/s}$, $v_{rms} = \sqrt{3RT/M} \approx 432 \text{ m/s}$.

Integrated problems

1: (i) $KE = (3/2)nRT = 1.5 \times 2 \times 8.31 \times 300 \approx 7.48 \times 10^3 \text{ J}$; (ii) $v_{rms} = \sqrt{3RT/M} \approx 484 \text{ m/s}$. 2: $c = \sqrt{3P/\rho} = \sqrt{3 \times 1.2 \times 10^5/1.6} = \sqrt{2.25 \times 10^5} \approx 474 \text{ m/s}$. 3: $v_H/v_O = \sqrt{M_O/M_H} = \sqrt{16} = 4$; $v_H = 4 \times 484 \approx 1.94 \times 10^3 \text{ m/s}$. 4: $v \propto \sqrt{T}$, $v'/v = \sqrt{T'/T} = \sqrt{1200/300} = 2$, $T' = 4 \times 300 = 1200 \text{ K} = 927^\circ \text{C}$. 5: $f = 3$, $C_V = 3R/2$, $C_P = 5R/2$, $\gamma = C_P/C_V = 5/3 \approx 1.67$.