

Limits and Derivatives — Assignment 2

Class 11 Maths · Higher-difficulty challenge set + integrated problems · Answer key included · Narayan Gurukul Academy

Instructions: (1) Attempt all questions; start with the topic-wise challenge set, then attempt the integrated (multi-topic) problems.
(2) Give all answers with correct SI units. $g = 10 \text{ m/s}^2$ unless stated.
(3) A final answer key is provided at the end — try the full set before checking it.

Part A — Topic-wise challenge questions

Intuitive Idea of a Limit

1. Evaluate $\lim_{x \rightarrow 2} (x^2 - 4)/(x - 2)$.
2. Evaluate $\lim_{x \rightarrow 0} (x^2 + 3x)/x$.
3. Evaluate $\lim_{x \rightarrow 2} (x^3 - 8)/(x^2 - 4)$.

Left-Hand and Right-Hand Limits

1. Find LHL and RHL of $f(x) = |x - 2|$ at $x = 2$.
2. For $f(x) = 1$ if $x < 0$, 2 if $x > 0$, find $\lim_{x \rightarrow 0} f(x)$.
3. Find LHL and RHL of $f(x) = x + 1$ for $x \geq 1$ and $x - 1$ for $x < 1$ at $x = 1$.

Algebra of Limits and Standard Limits

1. Evaluate $\lim_{x \rightarrow 0} (\tan x)/x$.
2. Evaluate $\lim_{x \rightarrow 0} (1 - \cos 2x)/x^2$.
3. Evaluate $\lim_{x \rightarrow 0} (1 - \cos 4x)/(1 - \cos 2x)$.

Continuity of a Function at a Point

1. Check the continuity of $f(x) = x^2 + 2$ at $x = 1$.
2. Check the continuity of $f(x) = |x|$ at $x = 0$.
3. Find k so that $f(x) = kx + 1$ for $x \leq 2$ and $3x - 5$ for $x > 2$ is continuous at $x = 2$.

Derivative as a Limit (First Principle)

1. Find the derivative of $f(x) = 3x + 2$ from first principles.
2. Find the derivative of $f(x) = x^3$ from first principles.
3. Find the derivative of $f(x) = \sqrt{x}$ from first principles.

Algebra of Derivatives

1. Differentiate $f(x) = x^3 + 2x^2 - 5x + 1$.
2. Differentiate $f(x) = x \sin x$ using the product rule.
3. Differentiate $f(x) = x/(x + 1)$ using the quotient rule.

Derivatives of Standard Functions

1. Differentiate $f(x) = 3x^4 - 2x^2 + 7$.
2. Differentiate $f(x) = 2 \sin x + 3 \cos x$.
3. Differentiate $f(x) = (x^2 + 1)/x$ using the quotient rule.

Part B — Integrated problems (multi-topic)

1. Evaluate $\lim_{x \rightarrow 2} (x^2 - 4)/(x - 2)$.
2. Evaluate $\lim_{x \rightarrow 0} (\sin x)/x$.
3. Find k so that $f(x) = kx + 1$ for $x \leq 2$ and $3x - 5$ for $x > 2$ is continuous at $x = 2$.
4. Find the derivative of $f(x) = x^3$ from first principles.
5. Differentiate $f(x) = x \sin x$ using the product rule.

Answer Key

Intuitive Idea of a Limit

4. 3. 3.

Left-Hand and Right-Hand Limits

LHL = RHL = 0 for $|x - 2|$ at $x = 2$. $\lim_{x \rightarrow 0} f(x)$ does not exist ($1 \neq 2$). LHL = 0, RHL = 2 at $x = 1$ for the piecewise function; limit does not exist.

Algebra of Limits and Standard Limits

1. 2. 4.

Continuity of a Function at a Point

$f(1) = 3 = \lim_{x \rightarrow 1} f(x)$ continuous. $|x|$ is continuous at 0. $kx + 1 \rightarrow 2k + 1$, $3x - 5 \rightarrow 1$; $2k + 1 = 1 \Rightarrow k = 0$.

Derivative as a Limit (First Principle)

3. $3x^2$. $1/(2\sqrt{x})$.

Algebra of Derivatives

$3x^2 + 4x - 5$. $\sin x + x \cos x$. $1/(x + 1)^2$.

Derivatives of Standard Functions

$12x^3 - 4x$. $2 \cos x - 3 \sin x$. $1 - 1/x^2$.

Integrated problems

- 1: 4. 2: 1. 3: $k = 0$. 4: $3x^2$. 5: $\sin x + x \cos x$.