

Oscillations — Assignment 2

Class 11 Physics · Higher-difficulty challenge set + integrated problems · Answer key included · Narayan Gurukul Academy

Instructions: (1) Attempt all questions; start with the topic-wise challenge set, then attempt the integrated (multi-topic) problems.
 (2) Give all answers with correct SI units. $g = 10 \text{ m/s}^2$ unless stated.
 (3) A final answer key is provided at the end — try the full set before checking it.

Part A — Topic-wise challenge questions

Periodic Motion and SHM Basics

1. Show that the motion of a mass attached to a spring is simple harmonic, and derive its time period.
2. Starting from the restoring force, derive the differential equation $x'' + \omega^2 x = 0$ and write its general solution.
3. A particle executes SHM with $x = 0.05 \sin(4\pi t)$ m. Find its amplitude, time period and maximum speed.

Spring-Mass System and Simple Pendulum

1. A mass of 0.5 kg is attached to a spring of spring constant 200 N/m. Find its period of oscillation.
2. The period of a simple pendulum is 2 s on the Earth. Find its period on the Moon ($g/6$).
3. Derive the time period of a simple pendulum for small oscillations. On what factors does it depend?

Energy in SHM

1. A particle of mass m executes SHM of amplitude A and angular frequency ω . Write its kinetic and potential energies as functions of displacement and show that the total energy is constant.
2. At what displacement is the kinetic energy of a SHM particle equal to its potential energy?
3. Derive $E = \frac{1}{2}m\omega^2 A^2$ and sketch how KE and PE vary with displacement.

Damped and Forced Oscillations

1. Define damped oscillations and resonance, and explain the difference between free, damped and forced oscillations.
2. Describe how the amplitude of a forced oscillator varies with the driving frequency, and explain resonance.
3. Give two practical examples of resonance and explain one hazard associated with it.

Part B — Integrated problems (multi-topic)

1. A 0.2 kg block on a horizontal frictionless surface is attached to a spring of force constant 80 N/m. It is pulled 5 cm and released. Find the angular frequency, time period and the maximum speed. Also write $x(t)$ if it starts from the extreme position.
2. A simple pendulum has a period of 2.0 s. Find its length ($g = 9.8 \text{ m/s}^2$). If the bob is taken to a lift accelerating upward at 1.8 m/s^2 , find the new period.
3. A particle executing SHM has an amplitude of 10 cm and a period of 2 s. Find its maximum velocity and maximum acceleration, and the velocity at a displacement of 5 cm.
4. Show that the total mechanical energy of a SHM oscillator of mass 0.1 kg, amplitude 0.05 m and angular frequency 10 rad s^{-1} is conserved, and compute its value in joules.

5. The amplitude of a lightly damped oscillator decreases to $1/e$ of its initial value in 20 s. Estimate the quality factor and explain what a large Q means for the sharpness of resonance.

Answer Key

Periodic Motion and SHM Basics

$a = -(k/m)x$ SHM with $T = 2\pi\sqrt{m/k}$. The differential equation $x'' + \omega^2 x = 0$ has solution $x = A \sin(\omega t + \phi)$. For $x = 0.05 \sin 4\pi t$: $A = 0.05$ m, $T = 0.5$ s, $v_{\max} = A\omega = 0.05 \times 4\pi \approx 0.63$ m s⁻¹.

Spring-Mass System and Simple Pendulum

$T = 2\pi\sqrt{0.5/200} \approx 0.31$ s. On the Moon, $T = \sqrt{6} \times 2 \approx 4.9$ s. $T = 2\pi\sqrt{l/g}$, depending only on l and g (not on mass or, for small swings, on amplitude).

Energy in SHM

$KE = \frac{1}{2}m\omega^2(A^2 - x^2)$, $PE = \frac{1}{2}m\omega^2 x^2$, $E = KE + PE = \frac{1}{2}m\omega^2 A^2 = \text{constant}$. $KE = PE$ at $x = A/\sqrt{2}$. Total energy $E = \frac{1}{2}m\omega^2 A^2$.

Damped and Forced Oscillations

Free: constant amplitude, no external force. Damped: amplitude falls exponentially (friction). Forced: driven by periodic force; at resonance (driving frequency = natural frequency) the amplitude is maximum. Examples: swing pushed at natural frequency; Tacoma Narrows bridge collapse (hazard).

Integrated problems

1: $\omega = \sqrt{k/m} = \sqrt{400} = 20$ rad s⁻¹; $T = \pi/10 \approx 0.31$ s; $v_{\max} = A\omega = 0.05 \times 20 = 1.0$ m s⁻¹; $x = 0.05 \cos 20t$. 2: $l = T^2 g / 4\pi^2 \approx 0.99$ m; effective $g = 9.8 + 1.8 = 11.6$ m/s² gives $T' \approx 1.84$ s. 3: $v_{\max} = A\omega = 0.1 \times \pi \approx 0.314$ m s⁻¹; $a_{\max} = A\omega^2 \approx 0.987$ m s⁻²; $v = \omega\sqrt{A^2 - x^2} \approx 0.272$ m s⁻¹. 4: $E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2} \times 0.1 \times 100 \times 0.0025 = 0.0125$ J. 5: $Q \approx \pi/\lambda$; amplitude $\rightarrow 1/e$ in 20 s gives a large Q — the resonance peak is sharp and the oscillator rings for a long time.