

Relations and Functions — Assignment 2

Class 11 Maths · Higher-difficulty challenge set + integrated problems · Answer key included · Narayan Gurukul Academy

Instructions: (1) Attempt all questions; start with the topic-wise challenge set, then attempt the integrated (multi-topic) problems.
 (2) Give all answers with correct SI units. $g = 10 \text{ m/s}^2$ unless stated.
 (3) A final answer key is provided at the end — try the full set before checking it.

Part A — Topic-wise challenge questions

Ordered Pairs and Cartesian Product

- If $A = \{1, 2\}$ and $B = \{3, 4\}$, find $A \times B$.
- If $A = \{1, 2, 3\}$ and $B = \{4, 5\}$, find $A \times B$ and $B \times A$.
- If $A = \{1, 2\}$, $B = \{3, 4\}$ and $C = \{5\}$, verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Relations

- Define the domain and range of a relation.
- If $A = \{1, 2, 3\}$ and $B = \{4, 5\}$, write two relations from A to B .
- If $A = \{1, 2, 3, 4, 5\}$, define $R = \{(a, b) : a + b = 6\}$. Write R in roster form and find its domain and range.

Functions and Their Graphs

- If $f(x) = 2x + 3$, find $f(0)$, $f(1)$ and $f(-1)$.
- Draw the graph of $f(x) = x^2$ for $x \in \mathbb{R}$ and state its range.
- Define a function and explain the domain, codomain and range with an example.

Types of Functions

- Show that $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 2x$ is injective.
- Explain surjective and bijective functions with one example each.
- Show that $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 3x + 2$ is a bijection.

Real Functions

- Find the domain of $f(x) = 1/(x^2 - 4)$.
- Find the domain and range of $f(x) = \sqrt{x - 1}$.
- Find the domain and range of $f(x) = 1/x$, $f(x) = \sqrt{x - 1}$ and $f(x) = |x|$.

Algebra of Real Functions

- If $f(x) = x + 2$ and $g(x) = 2x - 1$, find $(f + g)(x)$, $(f - g)(x)$ and $(fg)(x)$.
- If $f(x) = x + 1$ and $g(x) = x^2$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.
- If $f(x) = x + 1$ and $g(x) = 2x - 3$, find $(f + g)$, $(f - g)$, (fg) and (f/g) , stating the domain of (f/g) .

Part B — Integrated problems (multi-topic)

1. If $A = \{1, 2\}$ and $B = \{3, 4\}$, find $A \times B$ and $n(A \times B)$.
2. If $A = \{1, 2, 3, 4, 5\}$, define $R = \{(a, b) : a + b = 6\}$. Write R in roster form and find its domain and range.
3. Find the domain of $f(x) = 1/(x^2 - 4)$.
4. Show that $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 3x + 2$ is a bijection.
5. If $f(x) = x^2$ and $g(x) = x + 2$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.

Answer Key**Ordered Pairs and Cartesian Product**

$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$. $n(A \times B) = 6$; $B \times A = \{(4,1), (4,2), (4,3), (5,1), (5,2), (5,3)\}$. $A \times (B \cap C) = (A \times B) \cap (A \times C)$ holds.

Relations

Domain = first elements, range = second elements. $R = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$; domain = range = $\{1,2,3,4,5\}$.

Functions and Their Graphs

$f(0) = 3$, $f(1) = 5$, $f(-1) = 1$. Graph of x^2 is a parabola; range = $[0, \infty)$. A function assigns exactly one image to each element of the domain.

Types of Functions

$f(x) = 2x$ is injective since $2x_1 = 2x_2 \Rightarrow x_1 = x_2$. Surjective: range = codomain; bijective = both. $f(x) = 3x + 2$ is a bijection with inverse $(x - 2)/3$.

Real Functions

Domain of $1/(x^2 - 4) = \mathbb{R} - \{2, -2\}$. Domain of $\sqrt{x - 1} = [1, \infty)$, range = $[0, \infty)$. Domain/range of $1/x = \mathbb{R} - \{0\} / \mathbb{R} - \{0\}$; of $\sqrt{x-1}$ and $|x|$ as stated.

Algebra of Real Functions

$(f + g)(x) = 3x + 1$, $(f - g)(x) = -x + 3$, $(fg)(x) = 2x^2 + 3x - 2$. $(f \circ g)(x) = x^2 + 1$, $(g \circ f)(x) = (x + 1)^2$. $(f + g) = 3x - 2$; $(f - g) = -x + 4$; $(fg) = 2x^2 - x - 3$; $(f/g) = (x+1)/(2x-3)$, $x \neq 3/2$.

Integrated problems

1: $n(A \times B) = 4$. 2: R as above. 3: $\mathbb{R} - \{2, -2\}$. 4: $3x_1 + 2 = 3x_2 + 2 \Rightarrow x_1 = x_2$ (injective); for $y \in \mathbb{R}$, $x = (y - 2)/3 \in \mathbb{R}$ (surjective). 5: $(f \circ g)(x) = (x + 2)^2$; $(g \circ f)(x) = x^2 + 2$.