

Relations and Functions — Assignment 2

Class 12 Maths · Higher-difficulty challenge set + integrated problems · Answer key included · Narayan Gurukul Academy

Instructions: (1) Attempt all questions; start with the topic-wise challenge set, then attempt the integrated (multi-topic) problems.
 (2) Give all answers with correct SI units. $g = 10 \text{ m/s}^2$ unless stated.
 (3) A final answer key is provided at the end — try the full set before checking it.

Part A — Topic-wise challenge questions

Types of Relations

1. Show that the relation $R = \{(1, 1), (2, 2), (1, 2), (2, 1)\}$ on $A = \{1, 2\}$ is an equivalence relation.
2. Check whether the relation 'is less than' on the set of integers is reflexive, symmetric or transitive.
3. Show that the relation R on the set of integers defined by $R = \{(a, b) : a - b \text{ is divisible by } 5\}$ is an equivalence relation.

Types of Functions

1. Check whether $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 2x$ is one-one and onto.
2. Check whether $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = |x|$ is one-one.
3. Show that $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2$ is neither one-one nor onto.

Composition and Invertible Functions

1. If $f(x) = x + 2$ and $g(x) = 3x$, find $(f \circ g)(x)$.
2. Find the inverse of $f(x) = (x - 1)/2$.
3. Find the inverse of $f(x) = (3x + 2)/(x - 1)$, $x \neq 1$.

Binary Operations

1. Check whether the operation $a * b = a + b - 2$ on integers is commutative.
2. Find the identity element for the operation $a * b = a + b - 2$ on integers.
3. On the set of positive rationals, define $a * b = ab/2$. Find the identity element and check associativity.

Part B — Integrated problems (multi-topic)

1. Show that the relation $R = \{(1, 1), (2, 2), (1, 2), (2, 1)\}$ on $A = \{1, 2\}$ is an equivalence relation.
2. Show that $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3x + 2$ is bijective.
3. Find the inverse of $f(x) = 2x + 3$.
4. Check whether the operation $a * b = a + b - 2$ on integers is commutative.
5. Find the identity element for the operation $a * b = a + b - 2$ on integers.

Answer Key

Types of Relations

Reflexive, symmetric and transitive $\Rightarrow$ equivalence. 'Less than' is neither reflexive nor symmetric, but is transitive. Divisibility by 5: reflexive, symmetric, transitive $\Rightarrow$ equivalence.

Types of Functions

$f(x) = 2x$ is one-one and onto. $|x|$ is not one-one (-1 and 1 give 1). x^2 is neither one-one nor onto.

Composition and Invertible Functions

$(f \circ g)(x) = 3x + 2$. $f^{-1}(x) = 2x + 1$. For $f(x) = (3x + 2)/(x - 1)$: $f^{-1}(x) = (x + 2)/(x - 3)$.

Binary Operations

Commutative since $a + b - 2 = b + a - 2$. Identity e : $a + e - 2 = a \Rightarrow e = 2$. For $a * b = ab/2$: identity $e = 2$; associative.

Integrated problems

1: equivalence. 2: bijective (proof as in notes). 3: $(x - 3)/2$. 4: commutative. 5: $e = 2$.