

Sets — Assignment 2

Class 11 Maths · Higher-difficulty challenge set + integrated problems · Answer key included · Narayan Gurukul Academy

Instructions: (1) Attempt all questions; start with the topic-wise challenge set, then attempt the integrated (multi-topic) problems.
 (2) Give all answers with correct SI units. $g = 10 \text{ m/s}^2$ unless stated.
 (3) A final answer key is provided at the end — try the full set before checking it.

Part A — Topic-wise challenge questions

Sets and Their Representation

1. Write the set $\{2, 4, 6, 8\}$ in set-builder form.
2. Write the set of vowels of the English alphabet in roster form.
3. Write the set $\{1, 4, 9, 16, 25\}$ in set-builder form.

Types of Sets

1. Define an empty set with one example.
2. Write the power set of $A = \{a, b\}$.
3. Distinguish between equal sets and equivalent sets with examples.

Subsets, Power Set and Universal Set

1. List all the subsets of $A = \{1, 2, 3\}$ and identify the proper subsets.
2. If $n(A) = 3$, find $n(P(A))$.
3. Prove that if $A \subset B$ and $B \subset C$, then $A \subset C$.

Universal Set and Set Operations

1. If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, find $A \cup B$ and $A \cap B$.
2. If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$, find $A - B$ and $B - A$.
3. State and explain the commutative and associative laws of set operations with examples.

Venn Diagrams and De Morgan's Laws

1. Verify De Morgan's law $(A \cup B)' = A' \cap B'$ for $A = \{1, 2\}$, $B = \{2, 3\}$, $U = \{1, 2, 3, 4\}$.
2. Verify De Morgan's law $(A \cap B)' = A' \cup B'$ for $A = \{1, 2\}$, $B = \{2, 3\}$, $U = \{1, 2, 3, 4\}$.
3. State and prove De Morgan's laws using Venn diagrams.

Cardinality and Counting Problems

1. In a class of 40 students, 25 play cricket and 20 play football and 10 play both. Find how many play at least one game.
2. If $n(A) = 20$, $n(B) = 15$ and $n(A \cup B) = 30$, find $n(A \cap B)$.
3. In a survey of 100 people, 60 read newspaper A, 45 read newspaper B and 25 read both. Find how many read neither newspaper.

Part B — Integrated problems (multi-topic)

1. Write $A = \{x : x \text{ is a natural number, } x^2 < 20\}$ in roster form.
2. Find the power set of $A = \{1, 2, 3\}$ and state its cardinality.
3. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$ and $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, find $(A * B)'$.
4. Verify De Morgan's law $(A * B)' = A' \cap B'$ for $A = \{1, 2\}$, $B = \{2, 3\}$ and $U = \{1, 2, 3, 4\}$.
5. In a survey of 100 people, 60 read newspaper A, 45 read newspaper B and 25 read both. Find how many read neither.

Answer Key

Sets and Their Representation

$\{2, 4, 6, 8\} = \{x : x = 2n, n \in \mathbb{N}, n \leq 4\}$. Vowels = $\{a, e, i, o, u\}$. $\{1, 4, 9, 16, 25\} = \{x : x = n^2, n \in \mathbb{N}, n \leq 5\}$.

Types of Sets

Empty set has no elements, e.g. $\{x : x \in \mathbb{N}, x < 1\}$. Power set $P(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Equal sets have the same elements; equivalent sets have the same number of elements.

Subsets, Power Set and Universal Set

Subsets of $\{1, 2, 3\}$: $\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$; proper subsets exclude $\{1, 2, 3\}$. $n(P(A)) = 2^3 = 8$. If $A \subseteq B$ and $B \subseteq C$, every element of A is in B and hence in C, so $A \subseteq C$.

Universal Set and Set Operations

$A * B = \{1, 2, 3, 4\}$, $A \cap B = \{2, 3\}$. $A - B = \{1, 2\}$, $B - A = \{5\}$. Commutative: $A * B = B * A$; associative: $(A * B) * C = A * (B * C)$; similarly for $\cap$.

Venn Diagrams and De Morgan's Laws

$(A * B)' = \{4\} = A' \cap B'$. $(A \cap B)' = \{1, 3, 4\} = A' * B'$. Venn diagrams show the complement of the union is the intersection of the complements.

Cardinality and Counting Problems

$n(A * B) = 25 + 20 - 10 = 35$. $n(A \cap B) = 20 + 15 - 30 = 5$. Neither = $100 - (60 + 45 - 25) = 20$.

Integrated problems

1: $\{1, 2, 3, 4\}$ since $4^2 = 16 < 20$ and $5^2 = 25 > 20$. 2: $P = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$, $|P| = 8$. 3: $A * B = \{1, 2, 3, 4, 5, 6\}$, $(A * B)' = \{7, 8\}$. 4: $(A * B)' = \{4\} = A' \cap B'$. 5: 20 read neither.